Linear Algebra
Course booklet

Linear Algebra for college & university students. Contains vector calculus/spaces, matrices and matrix calculus, inner product spaces, and much more.

SOWISO

About SOWISO

SOWISO offers:

- a homework, practice and **learning environment**;
- personalised **feedback** on all answer attempts;
- different **testing and assessment** tools;
- customisable **mathematics courses** with explanations, examples, and endless **randomised practice exercises**;
- an authoring tool to **create original material**;
- **learning analytics** giving detailed insight into student performance;
- **integration** with your LMS/VLE.

Our learning environment guides students along as they solve problems. When doing exercises, students can enter open answer calculations or mathematical formulas. The software will analyse their answer and provide targeted feedback and hints helping the student understand the next step in the solution process, and/or highlight any mistakes they made.

SOWISO increases student engagement and saves teachers time checking and grading!

Pricing

SOWISO partners with higher education institutions on a SAAS licensing basis.

The cost for the platform starts at € 5.50 per student per year, with an additional per student per year fee of € 7.50 per course.

A second licensing model is one in which students pay for their own license in our webshop.

Our digital courses are a fully interactive alternative for paper books and offer a personalised and adaptive learning experience that fits today's generation of students.

How are courses structured?

The courses are structured in chapters and subchapters consisting of units. Every chapter starts with an introduction and ends with a conclusion The unit subjects are listed in more detail on the following pages.

Each unit consists of (at least) one theory page and one package of exercises.

Theory pages contain explanations, (randomized) examples and visualisations and (interactive) graphs.

The packages of **exercises** contain on average around 10 exercises. Each of these exercises are randomised, allowing for endless practicing, and include targeted hints and personalised feedback for the students while solving the exercises.

COURSE CONTENT

Chapter 1: Complex numbers (17 topics)

1. *Introduction to complex numbers (4 topics)*
    a. Imaginary numbers
    b. The notion of complex numbers
    c. Polar coordinates
    d. Real and imaginary part

2. *Calculating with complex numbers (4 topics)*
    a. Calculating with polar coordinates
    b. The quotient
    c. Complex conjugate
    d. Geometric interpretation

3. *Complex functions (4 topics)*
    a. Complex exponents
    b. Rules of calculation for complex powers
    c. Complex sine and cosine
    d. Complex logarithm

4. *Complex polynomials (5 topics)*
    a. The notion of a complex polynomial
    b. Factorization of complex polynomials
    c. Zeros of complex polynomials
    d. Fundamental theorem of algebra
    e. Real polynomials

Chapter 2: Vector calculus in plane and space (21 topics)

5. *Vectors in planes and space (4 topics)*
    a. The notion of vector
    b. Scalar multipliction
    c. Addition of vectors
    d. Linear combinations of vectors

6. *Straight lines and planes (2 topics)*
    a. Straight lines and planes
    b. Parametrization of a plane

7. *Bases, coordinates and equations (5 topics)*
    a. The notion of base
    b. Coordinate space
    c. Straight lines in the plane coordinates
    d. Planes in coordinate space
    e. Lines in the coordinate space

8. *Distances, angles and dot product (5 topics)*
    a. Distances, Angles and dot products
    b. Dot product
    c. Properties of the dot product
    d. The standard dot product
    e. Normal vectors

9. *The cross product (5 topics)*
    a. Cross product in 3 dimensions
    b. The concept of volume in space
    c. The volume of a parallelepiped
    d. Properties of cross product
    e. The standard cross product

Chapter 3: Systems of linear equations and matrices (21 topics)

10. *Linear equations (4 topics)*
    a. The notion of linear equation
    b. Reduction to a base form
    c. Solving a linear equation with a single unkown
    d. Solving a linaer equation with several unknowns

11. *Systems of linear equations (5 topics)*
    a. The notion of a system of linear equations
    b. Homogeneous and inhomogeneous systems
    c. Lines in the plane

17. *Coordinates (3 topics)*
    a. The notion of coordinates
    b. Coordinates of sums of scalar mulitples
    c. Basis and echelon form

Chapter 5: Inner product spaces (14 topics)

18. *Inner product, length, and angle (3 topics)*
    a. The notion of inner product
    b. Angle
    c. Perpendicularity

19. *Orthogonal systems (3 topics)*
    a. The notion of orthonormal system
    b. Properties of orthonormal systems
    c. Constucting orthonormal bases

20. *Orthogonal projections (3 topics)*
    a. Orthogonal projection
    b. Orthogonal complement
    c. Gram-Schmidt in matrix form

21. *Complex inner product spaces (5 topics)*
    a. Inner product on complex vector spaces
    b. Orthonormal systems in complex vector spaces
    c. Orthogonal complements in complex inner product spaces
    d. Complex orthogonal complements
    e. Gram-Schmidt in complex inner product spaces

Chapter 6: Linear maps (19 topics)

22. *Linear maps (9 topics)*
    a. The notion of linear map
    b. Linear maps determined by matrices
    c. Composition of linear maps
    d. Sums and multiples of linear maps
    e. The inverse of a linear map

f. Kernel and image of a linear transformation
g. Recording linear maps
h. Rank-nullity theorem for linear maps
i. invertibility criteria for linear maps

23. Matrices of linear maps (7 topics)

a. The matrix of a linear map in coordinate space
b. Determining the matrix in coordinate space
c. Coordinates
d. Basis transition
e. The matrix of a linear map
f. Coordinate transformations
g. Relationship to systems of linear equations

24. Dual vector spaces (3 topics)

a. The notion of dual space
b. Dual basis
c. Dual map

Chapter 7: Matrix calculus (16 topics)

25. Rank and inverse of a matrix (2 topics)

a. Rank and column space of a matrix
b. Invertibility and rank

26. Determinants (7 topics)

a. 2-dimensional determinants
b. Permutations
c. Higher-dimensional determinants
d. More properties of determinants
e. Row and column expansion
f. Row and column reduction
g. Cramer's rule

27. Matrices and coordinate transforms (4 topics)

a. Characteristic polynomial of a matrix
b. Conjugate matrices
c. Characteristic polynomial of a linear map

d. Matrix equivalence

28. *Minimal polynomial (3 topics)*
   a. Cayley-Hamilton
   b. Division with remainder for polynomials
   c. Minimal polynomial

Chapter 8: Invariant subspaces of linear maps (14 topics)

29. *Eigenvalues and eigenvectors (3 topics)*
   a. Diagonal form
   b. Eigenspace
   c. Determining eigenvalues and eigenvectors

30. *Diagonalizability (4 topics)*
   a. The notion of diagonalizability
   b. Diagonalizability and minimal polynomial
   c. The greatest common divisor of two polynomials
   d. The Euclidean algorithm

31. *Invariant subspaces (7 topics)*
   a. The notion of an invariant subspace
   b. The extended Euclidean algorithm
   c. Direct sum decomposition into invariant subspaces
   d. Generalized eigenspace
   e. Jordan normal form
   f. From real to complex vector spaces and back
   g. Real Jordan normal form for non-real eigenvalues

Chapter 9: Orthogonal and symmetric maps (17 topics)

32. *Orthogonal maps (5 topics)*
   a. The notion of orthogonal map
   b. Properties of orthogonal maps
   c. More properties of orthogonal maps
   d. Orthogonal matrices
   e. Orthogonal transformation matrices

33. *Classification of orthogonal maps (3 topics)*
  a. Low-dimensional orthogonal maps
  b. Jordan normal form for orthogonal maps
  c. Classification of orthogonal maps

34. *Unitary maps (2 topics)*
  a. The notion of unitary map
  b. Diagonal form for unitary maps

35. *Isometries (3 topics)*
  a. The notion of isometry
  b. Equivalence of isometries
  c. Characterisation of isometries

36. *Symmetric maps (4 topics)*
  a. The notion of symmetric map
  b. Connection with symmetric matrices
  c. Properties of symmetric maps
  d. Orthonormal bases and symmetric maps

37. *Application of symmetric maps (4 topics)*
  a. Quadratic forms
  b. Quadrics
  c. Least square solutions of linear equations
  d. Singular value decomposition

Chapter 10: Differential equations and Laplace transform (9 topics)

38. *Differential equations and Laplace transform (9 topics)*
  a. The Laplace transform
  b. The inverse Laplace transform
  c. Laplace transforms of differential equations
  d. Convolution
  e. Laplace transforms of Heaviside functions
  f. Laplace transforms of periodic functions
  g. Riemann-Stieltjes intergration
  h. Laplace transforms for delta function
  i. Transfer and response functions

Missing something? SOWISO allows teachers to create their own content in our authoring environment.

The notion of orthonormal system

Determine a function of the form $a\sqrt{x}+b$, where a and b are real numbers, such that the system of functions

$$\{1, \quad a \quad x \quad + b\}$$

is an orthonormal system in the inner product space of all functions on $[0,1]$ with inner product

$$f \cdot g = \int_0^1 f(x) \cdot g(x)\,\mathrm{d}x$$

Give your answer in the form of a linear combination of 1 and x .

Hint
Exress the inner products $1 \cdot (a\,x+b)$ and $(a\,x+b)\cdot(a\,x+b)$ in terms of a and b.

$x - \frac{1}{2}$ ⊗ The inner product of your answer with the constant function 1 is indeed equal to 0, but the length of your answer is not equal to 1.

Hint
The dot products from the first hint are respectively equal to 0 and 1 because of the required orthonormality. This gives two equations with unknowns a and b.

$2\sqrt{3}\cdot x - \sqrt{3}$ ✓ Great

Practice example

From systems to matrices

The *elimination method*, which solves systems of linear equations by use of elementary operations, actually works only with the coefficients and constants of the system. A good accounting in the form of a succinct notation can help expedite this process.

Definition

The system of m linear equations with n unknowns $x_1, \ldots, x_n$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

is often written as follows:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

Such a rectangular array is called a **matrix**, and is often framed in round brackets. Since the unknowns $x_1, \ldots, x_n$ and their order of appearance does not change during the solving process, the system is also well represented by the matrix

$$\left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \end{array}\right.$$

Theory example

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